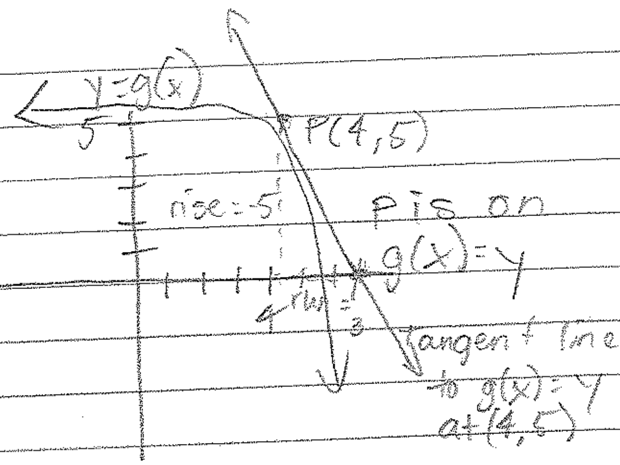
2.1

#43 $y = g(x)$



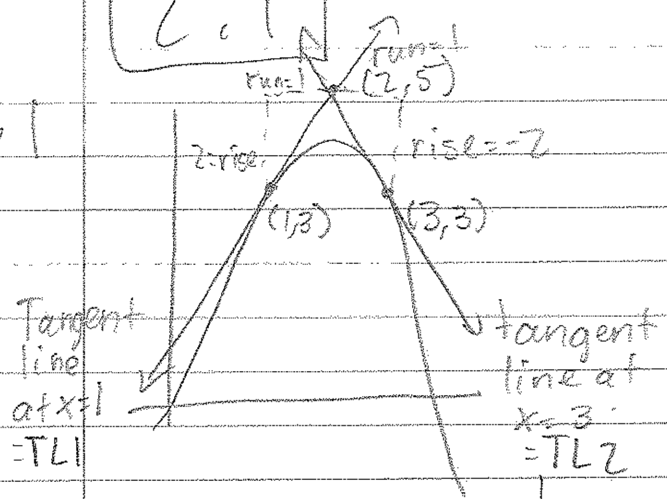
a) find $\boxed{g(4) = 5}$
and
b) find $g'(4)$ = slope of tangent line at (4,5)

$= m_{tan}\Big|_{x=4} = \dfrac{\text{rise}}{\text{run}}$

$$\boxed{g'(4) = \frac{-5}{3}}$$

2.1

#61



$m_{tan}\Big|_{x=1} = \frac{2}{1} = 2$

$m_{tan}\Big|_{x=3} = \frac{-2}{1} = -2$

<u>TL_1 equation</u>

$y - y_1 = m_{tan}(x - x_1)$

$y - (3) = 2(x - 1)$

$3 + y - 3 = 2x - 2 + 3$

$\boxed{y = 2x + 1}$

<u>TL_2 equation</u>

$y - y_1 = m_{tan}(x - x_1)$

$y - 3 = -2(x - 3)$

$3 + y - 3 = -2x + 6 + 3$

$\boxed{y = -2x + 9}$

#61 finding the derivative using limits

$f(x) = 4x - x^2$

$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{[4(x+\Delta x) - (x+\Delta x)^2] - [4x - x^2]}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\cancel{4x} + 4\Delta x - \cancel{x^2} - 2x\Delta x - (\Delta x)^2 - \cancel{4x} + \cancel{x^2}}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{4\Delta x - 2x\Delta x - \Delta x^2}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(4 - 2x - \Delta x)}{\cancel{\Delta x}}$

$f'(x) = \lim_{\Delta x \to 0} 4 - 2x - \Delta x$

$f'(x) = 4 - 2x - (0)$

$f'(x) = 4 - 2x$

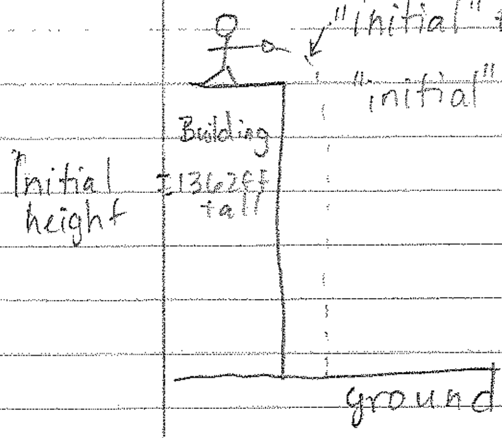
$f'(1) = 4 - 2(1)$ $f'(1) = 2$ This is the slope of TL_1	$f'(3) = 4 - 2(3)$ $f'(3) = 4 - 6$ $f'(3) = -2$ This is the slope of TL_2

$\boxed{2.2}$

#97 A silver dollar is dropped from the top of a building that is 1362 ft tall.

"initial" time → $t = 0$

"initial" height → $S_0 = 1{,}362$ ft



a) Position function:

$s(t) = -16t^2 + V_0 t + S_0$

V_0 = initial velocity = $0 \frac{ft}{sec}$ (because it was dropped)

S_0 = initial position = 1,362 ft (building height)

$s(t) = -16t^2 + 0t + 1362$

$\boxed{s(t) = -16t^2 + 1362}$

Velocity function:

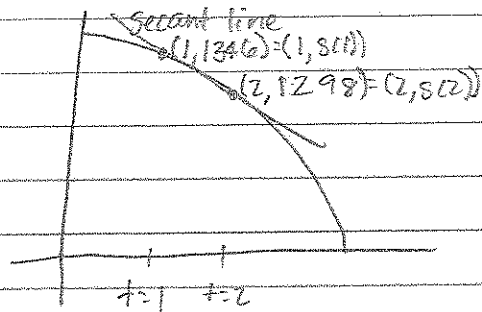
$v(t) = \frac{d}{dt}[s(t)] = s'(t)$

$v(t) = \frac{d}{dt}[-16t^2 + 1362]$

$v(t) = -16 \cdot 2t + 0$

$\boxed{v(t) = -32t}$

b) avg velocity on the interval [1, 2]



avg velocity on [1, 2] $= \frac{\Delta s}{\Delta t} = \frac{s(2) - s(1)}{2 - 1} = \frac{1298 - 1346}{1} = \boxed{\frac{-48 \text{ ft}}{\text{sec}}}$

c) instantaneous velocities when $t = 1$ and $t = 2$

$V(1) = -32(1)$

$\boxed{V(1) = -32 \frac{\text{ft}}{\text{sec}}}$

$V(2) = -32(2)$

$\boxed{V(1) = -64 \frac{\text{ft}}{\text{sec}}}$

d) time required for coin to reach ground level

s(t) = position

s(t) = 0 ft (ground level)

$-1362 + 0 = -16t^2 + 1362 + (-1362)$

$\left(\frac{1}{-16}\right) 1362 = -16t^2 \left(\frac{1}{-16}\right)$

$\sqrt{\frac{681}{8}} = \sqrt{t^2}$

$t = \sqrt{\frac{681}{8}} = \frac{\sqrt{681}}{\sqrt{4} \cdot \sqrt{2}} = \frac{\sqrt{681}}{2\sqrt{2}} \left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \boxed{\frac{\sqrt{1362}}{2} \text{ sec}}$

e) velocity of coin at impact

impacts at $\frac{\sqrt{1362}}{4}$ seconds

$V\left(\frac{\sqrt{1362}}{2}\right) = -32\left(\frac{\sqrt{1362}}{2}\right)$

$= \boxed{-16\sqrt{1362}\ \frac{ft}{sec}}$

$\boxed{2.2}$

#67 $f(x) = \frac{K}{x}$ $\quad y = -\frac{3}{4}x + 3$ is tangent to $f(x)$

$f'(c) = m_{tan} = -\frac{3}{4} \Big|_{x=c}$

$f(x) = Kx^{-1}$

$f'(x) = \frac{d}{dx}[Kx^{-1}]$

$f'(x) = K \cdot (-1x^{-2})$

$f'(x) = \frac{-K}{x^2}$

Tangent line at $(c, f(c))$

$y - f(c) = m_{tan}(x - c)$

$y = m_{tan}x \underbrace{- c \cdot m_{tan} + f(c)}$

$y = \underset{\text{I}}{\frac{-3}{4}}x + \underset{\text{II}}{3}$

at $x = c$, $f(c) = ?$

$f(c) = \frac{K}{c}$

I $f'(c) = \frac{-K}{c^2} \rightarrow \frac{-3}{4} = \frac{-K}{c^2}$

$\boxed{\frac{3}{4} = \frac{K}{c^2}}$

II $3 = -c\left(\frac{-3}{4}\right) + \frac{K}{c}$

$\boxed{3 = \frac{3}{4}c + \frac{K}{c}}$

I $\frac{3}{4} = \frac{K}{c^2}$ $\quad$ II $3 = \frac{3}{4}c + \frac{K}{c}$

substitute $\boxed{\frac{3c^2}{4} = K}$ $\quad$ next page

II $3 = \frac{3}{4}c + \frac{k}{c}$

$3 = \frac{3}{4}c + \frac{(\frac{3}{4}c^2)}{c}$

$3 = \frac{3}{4}c + \frac{3}{4}c$

$3 = \frac{6}{4}c$

$(\frac{2}{3})3 = \frac{3}{2}c(\frac{2}{3})$

$2 = c$

if $C = 2$ then

$\frac{3}{4}(2)^2 = k$

$\boxed{3 = K}$ Holy cow!

$\boxed{2.2}$

\#65 $f(x) = x^2 - kx$ tangent line: $5x - 4$

$f'(x) = \frac{d}{dx}[x^2 - kx]$ at $x = c$, $f(c) = c^2 - kc$

$f'(x) = 2x - 2$

at $x = c$, $f'(c) = 2c - k$

$f'(c) = 5$

I $5 = 2c - k$

II $-y = -5c + c^2 - kc$

$y - f(c) = m_{tan}(x - c)$

$y = \underset{\downarrow}{m_{tan}}\, x \underbrace{- c \cdot m_{tan} + f(c)}$

$m_{tan} = 5$
$f'(c) = 5$

$-c \cdot m_{tan} + f(c)$
$-c \cdot 5 + c^2 - kc$

I $2c - k = 5$
$2c - k + k = k + 5$
$-5 + 2c = k + 5 - 5$
$2c - 5 = k$

II $-y = -5c + c^2 - kc$
$-y = -5c + c^2 - (2c - 5)$
$-y = \cancel{-5c} + \cancel{c^2} - 2\cancel{c^2} + \cancel{5c}$
$(-1) - y = -c^2 (-1)$
$y = c^2$

either

$c = 2$ or $c = -2$

i Find K, $c = 2$

$2c - 5 = K$ So, $f(x) = x^2 - kx$ $f'(x) = 2x + 1$
$2(2)5 = K$ $f(x) = x^2 + x$ $f(2) = 5$
$-1 = K$

ii find k, $c=-2$

$2c-5=k$
$2(-2)-5=k$
$-9=k$

So, $f(x)=x^2-kx$
$f(x)=x^2+9x$

$f'(x)=2x+9$
$f'(-2)=5?$
$f'(-2)=2(-2)+9$
$f'(-2)=5$

either
$\boxed{K=-9 \text{ or } k=-1}$

$\boxed{2.2}$

#113 $y = ax^2 + bx + c$

$\frac{d}{dx}[y] = \frac{d}{dx}[ax^2 + bx + c]$

$\frac{dy}{dx} = a \cdot 2x + b + c$

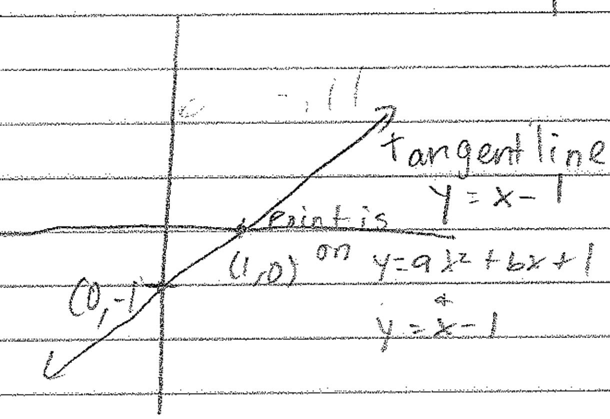
$\frac{dy}{dx} = 2ax + b$

$y = ax^2 + bx + c$ passes through $(0,1)$

$y = a(0)^2 + b(0) + c = 1$

$\boxed{c = 1}$

So, $y = ax^2 + bx + 1$
find a & b



at $(1,0)$
$m_{tan}\big|_{x=1} = 1$

$\frac{dy}{dx}\Big|_{x=1} = 1$

$2a(1) + b = 1$

I $2a + b = 1$

Since $(1,0)$ is on $y = ax^2 + bx + 1$
$y = 0$ when $x = 1$

III $1+0=a(1)^2+b(1)+1+(-1)$

$1=a+b$

Solve:

$-(a+b)=(-1)(-1)$

$2a+b=1$

$2a+b=1$

$-a-b=1$

$a=2$

$a+b=-1$

$(2)+b=-1$

$-2+2+b=-1+(-2)$

$b=-3$

$$\boxed{y=2x^2-3x+1}$$

$\boxed{2.3}$

#81 a) find $p'(1)$

$p(x) = f(x)g(x)$

$\frac{d}{dx}[p(x)] = p'(x) = \frac{d}{dx}[f(x) \cdot g(x)]$

$= (g(x))\frac{d}{dx}(f(x)) + (f(x))\frac{d}{dx}(g(x))$

$p'(x) = g(x)f'(x) + f(x)g'(x)$

$p'(1) = g(1)f'(1) + f(1)g'(1)$

$p'(1) = (4)(1) + (6)(-\frac{1}{2})$

$p'(1) = 4 + (-3)$

$\boxed{p'(1) = 1}$

$g(1) = 4$
$f(1) = 6$
$f'(1) = 1$ slope of tangent line when $x=1$
$g'(1)$

$\boxed{2.3}$

#75 $f(x) = \dfrac{x^2}{x-1}$ find horizontal tangent lines.

$$\frac{d}{dx}[f(x)] = \frac{d}{dx}\left[\frac{x^2}{x-1}\right]$$

$$f'(x) = \frac{(x-1)\frac{d}{dx}(x^2) - (x^2)\frac{d}{dx}(x-1)}{(x-1)^2}$$

$$f'(x) = \frac{(x-1)(2x) - (x^2)(1)}{(x-1)^2}$$

$$f'(x) = \frac{x[2(x-1) - x]}{(x-1)^2}$$

$$f'(x) = \frac{x(2x-2-x)}{(x-1)^2}$$

$$f'(x) = \frac{x(x-2)}{(x-1)^2}$$

$f'(x) = 0$ when we have horizontal tangents

$f'(x) = 0$

$$\frac{x(x-2)}{(x-1)^2} = 0$$

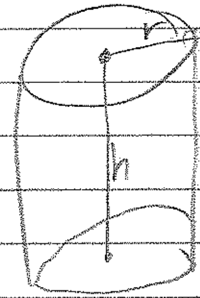
$x(x-2) = 0$

either

$x=0$ or $x=2$

$\boxed{2.3}$

#84 radius $= \sqrt{t+2}$ in
height $= \frac{1}{2}\sqrt{t}$ in
t = time in seconds



Volume of cylinder =
$\pi r^2 h$

$$V = \pi\left(\sqrt{t+2}\right)^2\left(\tfrac{1}{2}\sqrt{t}\right)$$
$$= \frac{\pi}{2}(t+2)\sqrt{2}$$
$$= \frac{\pi}{2}(t+2)t^{1/2}$$
$$= \frac{\pi}{2}\left(t^{3/2}+2t^{1/2}\right)$$

$$\frac{dV}{dt} = \frac{d}{dt}\left[\frac{\pi}{2}\left(t^{3/2}+2t^{1/2}\right)\right]$$

$$\frac{dV}{dt} = \frac{\pi}{2}\cdot\frac{d}{dt}\left[t^{3/2}+2t^{1/2}\right]$$

$$\frac{dV}{dt} = \frac{\pi}{2}\left[\frac{3}{2}t^{1/2}+2\cdot\frac{1}{2}t^{-1/2}\right]$$

$$\frac{dV}{dt} = \frac{\pi}{2}\left[\frac{3}{2}t^{1/2}+t^{-1/2}\right]$$

$$\frac{dV}{dt} = \frac{\pi}{2}\left[t^{-1/2}\left(\frac{3}{2}t+1\right)\right]$$

$$\frac{dV}{dt} = \frac{\pi\left(\frac{3}{2}t+1\right)}{2t^{1/2}}\left(\frac{2}{2}\right)$$

$$\boxed{\frac{dV}{dt} = \frac{\pi(3t+2)}{4\sqrt{t}}\ \frac{in^3}{sec}}$$

$\boxed{2.4}$

#31 $f(v) = \left(\frac{1-2v}{1+v}\right)^3$

Find the derivative

$$f'(v) = \frac{d}{dv}\left[\left(\frac{1-2v}{1+v}\right)^3\right]$$

$$f'(v) = 3\left(\frac{1-2v}{1+v}\right)^2 \cdot \frac{d}{dv}\left(\frac{1-2v}{1+v}\right)$$

$$f'(v) = 3\left(\frac{1-2v}{1+v}\right)^2 \cdot \left[\frac{(1+v)\frac{d}{dv}(1-2v) - (1-2v)\frac{d}{dv}(1+v)}{(1+v)^2}\right]$$

$$f'(v) = 3\left(\frac{1-2v}{1+v}\right)^2 \cdot \left[\frac{(1+v)(-2) - (1-2v)(1)}{(1+v)^2}\right]$$

$$f'(v) = 3\left(\frac{1-2v}{1+v}\right)^2 \cdot \left[\frac{-2 - \cancel{2v} - 1 + \cancel{2v}}{(1+v)^2}\right]$$

$$f'(v) = 3\left(\frac{1-2v}{1+v}\right)^2 \cdot \left[\frac{-3}{(1+v)^2}\right]$$

$$\boxed{f'(v) = \frac{-9(1-2v)^2}{(1+v)^4}}$$

$\boxed{2.4}$

#62 $y = 3x - 5\cos[(\pi x)^2]$

Find the derivative

$\frac{d}{dx}[y] = \frac{d}{dx}[3x - 5\cos[(\pi x)^2]$

$y' = \frac{d}{dx}[3x] - \frac{d}{dx}[5\cos[(\pi x)^2]$

$y' = 3\cdot(1) - [5 \cdot (-\sin(\pi x)^2)\frac{d}{dx}((\pi x)^2)]$

$y' = 3 - [-5\sin(\pi x)^2 \cdot (2\pi x)\frac{d}{dx}(\pi x)]$

$y' = 3 - [-5\sin(\pi x)^2 \cdot (2\pi x)(\pi)]$ ☆☆☆

$y' = 3 + 10\pi^2 x(\sin[(\pi x)^2]$

$\boxed{2.4}$

#64 $y = \sin \sqrt[3]{x} + \sqrt[3]{\sin x}$

$y = \sin(x^{1/3}) + \sin^{1/3}(x)$

find the derivative

$\frac{d}{dx}[y] = \frac{d}{dx}[\sin(x^{1/3}) + [(\sin x)^{1/3}]]$

$y' = (\cos(x^{1/3}))\frac{d}{dx}(x^{1/3}) + (\frac{1}{3}(\sin x)^{-2/3})\frac{d}{dx}(\sin x)$

$\boxed{y' = (\cos(x^{1/3}))(\frac{1}{3}x^{-2/3}) + \frac{1}{3}(\sin x)^{-2/3}(\cos x)}$

$\boxed{2.4}$

#107 c) $r(x) = f(-3x)$

$\frac{d}{dx}[r(x)] = \frac{d}{dx}[f(-3x)]$

$r'(x) = (f'(-3x))\frac{d}{dx}(-3x)$

$r'(x) = (f'(-3x))(-3)$

$r'(-2)$

$x = -2$

$-3x = -3(-2)$
$x = 6$

$\boxed{r'(-2) = f'(6)\cdot(-3)}$

$\boxed{2.5}$

#13 $\sin(x) = x(1 + \tan y)$

$\sin(x) = x + x(\tan(y))$

find $\frac{dy}{dx}$

$\frac{d}{dx}[\sin(x)] = \frac{d}{dx}[x + x(\tan(y))]$

$\cos(x) = 1 + [(x)\frac{d}{dx}(\tan(y)) + (\tan(y))\frac{d}{dx}(x)]$

$\cos(x) = 1 + [(x)(\sec^2(y))\frac{d}{dx}(y) + (\tan(y))(1)]$

$\cos(x) = 1 + [(x)(\sec^2(y))(\frac{dy}{dx}) + (\tan(y))]$

$(-1 - \tan(y)) + \cos(x) = -1 - \tan(y) + 1 + \tan(y) + x\sec^2(y)\frac{dy}{dx}$

$\frac{-1 - \tan(y) + \cos(x)}{x\sec^2(y)} = x\sec^2(y)\frac{dy}{dx}\left(\frac{1}{x\sec^2(y)}\right)$

$$\boxed{\frac{-1 - \tan(y) + \cos(x)}{x\sec^2(y)} = \frac{dy}{dx}}$$

$\boxed{2.5}$

#57 $25x^2 + 16y^2 + 200x - 160y + 400 = 0$

find the points where the graph has horizontal and vertical tangent lines

$\frac{d}{dx}[25x^2 + 16y^2 + 200x - 160y + 400] = \frac{d}{dx}[0]$

$25 \cdot \frac{d}{dx}(x^2) + 16 \cdot \frac{d}{dx}(y^2) + 200 \cdot \frac{d}{dx}(x) - 160 \cdot \frac{d}{dx}(y) + 0 = 0$

$25(2x) + 16 \cdot 2y \cdot \frac{dy}{dx} + 200(1) - 160\frac{dy}{dx} = 0$

$-50x - 200 + 50x + 200 + 32y\frac{dy}{dx} - 160\frac{dy}{dx} = 0 - 50x - 200$

$32y\frac{dy}{dx} - 160\frac{dy}{dx} = -50x - 200$

$\frac{1}{32y-160} \cdot \frac{dy}{dx}[32y - 160] = \frac{-50x - 200}{32y - 160}$

$\frac{dy}{dx} = \frac{-50(x+y)}{32(y-5)}$

$\frac{dy}{dx} = \frac{25(x+y)}{-16(y-5)}$

① Horizontal tangent lines

$\frac{dy}{dx} = 0$

$\frac{-25(x+4)}{16(y-5)} = 0$

$x + 4 = 0$

$x = -4$

Ⅱ Vertical tangent lines

$\frac{dy}{dx} = \text{undefined} = \frac{\text{something}}{0}$

$16(y-5) = 0$

$y - 5 = 0$

$y = 5$